Factor Analysis for Volatility - Part II

Ross Askanazi and Jacob Warren

September 4, 2015

Review - Intro

- Canonical factor model:

$$y_t = \beta F_t + e_t$$

- Induces the convenient decomposition on the covariance matrix:

$$\Sigma^y = \beta \Sigma^f \beta' + \Sigma^e$$

  Where Σ^e is likely diagonal (or at least sparse)

- And we can make any of the above three objects time-varying to produce time-varying covariances

Review - Literature

- Classical models (Diebold and Nerlove, 1989, many others):

$$\Sigma_t^y = \beta \Sigma_t^f \beta' + \Sigma^e$$
$$\log(\Sigma_t^f) = \alpha^f \log(\Sigma_{t-1}^f) + u_t^f$$

- Simple extensions (Pitt and Shephard, 1999, many others):

$$\Sigma_t^y = \beta \Sigma_t^f \beta' + \Sigma_t^e$$
$$\log(\Sigma_t^f) = \alpha^f \log(\Sigma_{t-1}^f) + u_t^f$$
$$\log(\sigma_{t,i}^e) = \alpha_i^e \log(\sigma_{t-1,i}^e) + u_{t,i}^e$$

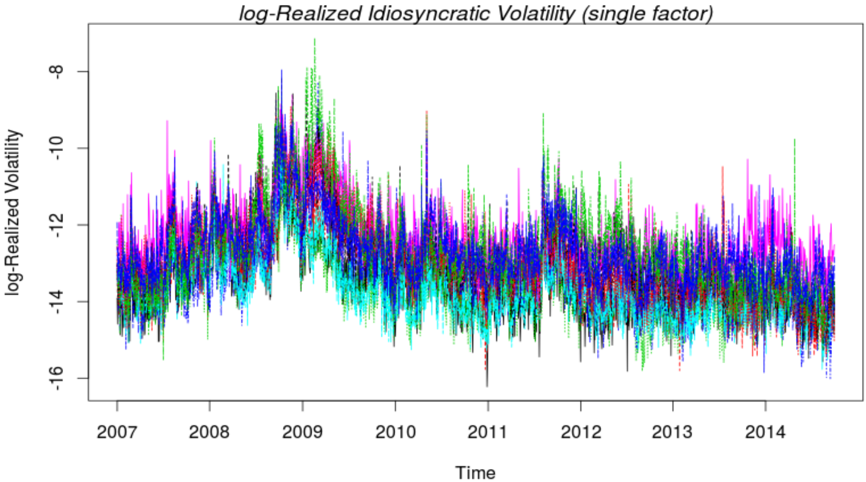
Review - Literature

- Factor for Volatility (Barigozzi and Hallin (2015), Herskovic, Kelly, et al (2014), growing literature)

$$
\begin{aligned}
\Sigma_t^y &= \beta \Sigma_t^f \beta' + \Sigma_t^e \\
\log(\Sigma_t^f) &= \beta^f V_t + u_t^f \\
\log(\sigma_{t,i}^e) &= \beta_i^e V_t + u_{t,i}^e \\
V_t &= \beta^v V_{t-1} + u_t^v
\end{aligned}
$$

Review - Empirics

- Fit factor model at high frequency using intraday returns
- Record realized variance of factor and idiosyncratic error



Conditional Mean Dynamics

- "If we want to have any hope capturing conditional variance dynamics, we need to be sure of the conditional mean dynamics first."
- Could the observed comovement between factor and idiosyncratic volatilities be due to omitted conditional mean dynamics?

Quadratic factor

If the true DGP is:

$$y_t = \beta_1 f_t + \beta_2(f_t^2 - \sigma_{f_t}^2) + e_t$$
$$f_t \sim N(0, \sigma_{f,t}^2)$$

Yet the estimated model is:

$$y_t = \bar{\beta}_1 f_t + \bar{e}_t$$

Then:

$$\mathbb{E}[\bar{\beta}_1] = \beta_1 + \beta_2 \frac{Cov(f_t, f_t^2)}{\mathbb{V}[f_t]} = \beta_1 \quad \text{under symmetry} \tag{1}$$
$$\bar{e}_t = \beta_2(f_t^2 - \sigma_{f_t}^2) + e_t \tag{2}$$
$$\mathbb{V}_t[\bar{e}_t] = 2\sigma_{f_t}^4 \beta_2 \beta_2' + \mathbb{V}_t[e_t] \tag{3}$$

Even if $\mathbb{V}_t[e_t] = c$, $\mathbb{V}_t[\bar{e}_t]$ will be time-varying and comove with market volatility!

White's Theorem and Cubic factor

White's Theorem: *Any* nonlinear model can be well-approximated by a time-varying parameter linear model.

Although the research is inconclusive, some work (including ours) show that βs vary over time

Could the time-variation also be a result of ommited variables?

Cubic Factor

Now let the DGP be:

$$y_t = \beta_1 f_t + \beta_2 (f_t^2 - \sigma_{f_t}^2) + \beta_3 f_t^3 + e_t$$

and we fit a misspecified linear model with time-varying coefficients:

$$y_t = \bar{\beta}_{1,t} f_t + \bar{e}_t, \quad \bar{e}_t \sim N(0, \Sigma_t)$$

Then:

$$\bar{\beta}_{1,t} = \frac{cov_t(y_t, f_t)}{\mathbb{V}_t[f_t]} = \beta_1 + \beta_3 \frac{cov_t(f_t^3, f_t)}{\mathbb{V}_t[f_t]} = \beta_1 + 3\beta_3 \sigma_{f_t}^2$$

Time-varying βs with factor structure!

This is White's Theorem in action, since the time-varying parameters pick up the nonlinearities.

Cubic Factor (Cont'd)

Looking at the residuals:

$$\bar{e}_t = \beta_2(f_t^2 - \sigma_{f_t}^2) + \beta_3 f_t^3 - 3\beta_3\sigma_{f_t}^2 f_t + e_t \quad (4)$$

$$\mathbb{V}_t[\bar{e}_t] = 2\sigma_{f_t}^4 \beta_2\beta_2' + (9\sigma_{f,t}^4 - 3\sigma_{f,t}^6)\beta_3\beta_3' + \mathbb{V}_t[e_t] \quad (5)$$

And once again, we get a factor structure on volatility, even if $\mathbb{V}_t[e_t]$ is actually constant!

Leverage Effects

- "Leverage Effects" are the phenomenon that there is general negative correlation between an asset return and its (changes in) volatility
  - The story is that a price decrease results in a more leveraged position, since the value of debt rises relative to that of equity
  - Black (1976) and Christie (1982)
- "Risk Premia Effects" are the phenomenon that higher returns should be positively correlated with risk
  - Pindyck (1984) and French, Schwert and Stambaugh (1987)

For a given stock return, r_t, this corresponds to:

$$\log\left(\frac{\sigma_{t+1}}{\sigma_t}\right) = \alpha + \lambda_0 r_t + \varepsilon_{t+1,0}$$

Where λ_0 would be negative (leverage) or positive (risk premia).

Leverage Effects

Duffee (1995) finds that in fact the above formula is misspecified, and should be re-written as:

$$\log(\sigma_t) = \alpha_1 + \lambda_1 r_t + \varepsilon_{t,1}$$
$$\log(\sigma_{t+1}) = \alpha_2 + \lambda_2 r_t + \varepsilon_{t+1,2}$$

Where $\hat{\lambda}_1 > \hat{\lambda}_2$, which creates the perceived leverage effect.

He then estimates the model in a factor context and finds:

$$\log(\sigma_{f,t}) = \lambda_f f_t + \Phi(L)\sigma_{f,t} + v_{m,t}$$
$$\log(\sigma^i_{e,t}) = \lambda_i f_t + \Phi(L)\sigma^i_{e,t} v_{i,t} \tag{6}$$

that $\lambda_f < 0$ and $\lambda_i > 0$
Though again, there is no consensus about the sign of the coefficients.

Leverage Effects

Can our omitted variable setup explain this puzzle?

In the case of a missing square term,
Let $\lambda^{\star} = Cov(f_t, \sigma^2_{f,t}) \neq 0$, so

$$\mathbb{E}[\bar{\beta}_1] = \beta_1 + \bar{\lambda}\beta_2 \tag{7}$$

Then the error term is:

$$\bar{e}_t = \beta_2(f_t - \sigma_{f,t}) - \bar{\lambda}\beta_2 f_t + e_t \tag{8}$$

$$\mathbb{V}_t[\bar{e}_t] = g(\bar{\lambda}, \beta_2, \dots)\sigma^2_{f,t} + others \tag{9}$$

$$= \bar{g}(\bar{\lambda}, \beta_2, \dots) f_t + others \qquad \text{by eqn 6} \tag{10}$$

Nonzero correlation between idiosyncratic variance and market returns!

Data

Are there actually higher-order dynamics?

Data

Are there actually higher-order dynamics?

Fit fourth-order polynomial to daily DOW-10 returns with SPY as factor from Jan 2007 - Sep 2014

	Polynomial Order				
	2	3	4	R^2	F-test
1	-	-	**	0.008	5.492
2	-	***	***	0.043	29.02
3	-	-	**	0.015	9.709
4	-	***	-	0.013	8.228
5	***	-	***	0.092	65.795
6	***	-	***	0.016	10.412
7	*	-	*	0.002	1.272
8	***	*	***	0.078	55.178
9	-	-	**	0.007	4.549
10	***	-	***	0.031	20.722

Data

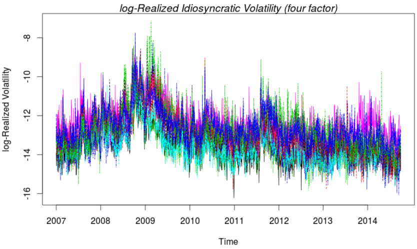
Fit fourth-order polynomial factor model intraday to DOW-10 with observed SPY factor.

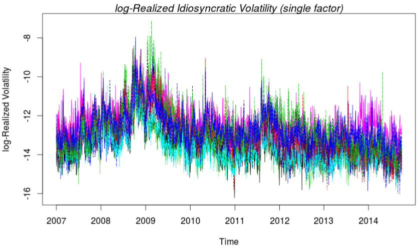
Extract residual volatility:

Data

Fit fourth-order polynomial factor model intraday to DOW-10 with observed SPY factor.

Extract residual volatility:



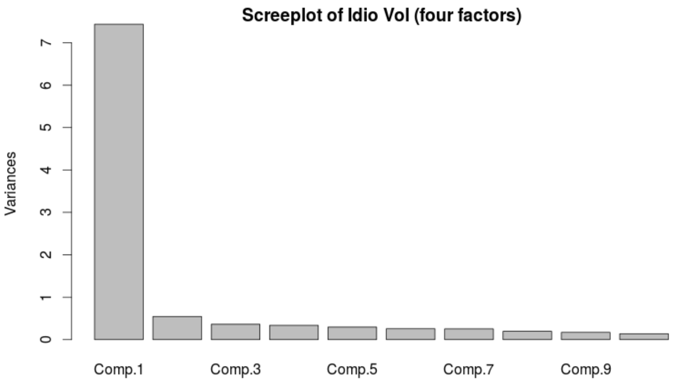


Screeplot

Final Thoughts

- At the end of the day, the units in idiosyncratic volatility are very small.
- From a forecasting perspective, are we better off just holding them constant?
- Regime switching with two regimes?

Nonlinear Screeplot



Back